# MODELING AND SIMULATION OF HYDRAULIC SYSTEM FOR TRANSLATIONAL AND ROTATIONAL MOTION

**Željko Šitum, Mario Essert, Tihomir Žilić**

University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
10000 Zagreb, Ivana Lučića 5, Croatia

*zeljko.situm@fsb.hr (Željko Šitum)*

**Abstract**

The main focus in this paper is on modeling and computer simulations of hydraulic systems for achieving a linear or rotary actuator motion, which are mostly used for high-power industrial applications. The hydraulic servo systems are very important systems for control applications because they take the advantages of both the large output power of traditional hydraulic systems and a high servo control response of electric systems. A nonlinear dynamic model for both translational and rotational motion of hydraulic actuator has been derived, which can be used in simulation studies of dynamic behavior of system and for advanced controller design. The hydraulic control system model has been derived as coupled sets of nonlinear algebraic and differential equations, which includes the main phenomena of physical system. This form of modeling equations has been resulted in a block-oriented system model. The simulation model includes the flow and force equations necessary to describe dynamic behavior of the control system and allows the prediction of performance of hydraulic system through computer simulation. A simulation model depicting the two control systems has been built, which may be used for an easy comparison of dynamical behavior of common hydraulic systems in industry. The model also contains several important dynamic characteristics those substantially influence the performance of the system within the operation range. Model development and preliminary experimental testing on laboratory setup are in progress.

**Keywords: Hydraulic system, Mathematical model, System simulation**

### Presenting Author's biography

Željko Šitum. Received the BS degree in 1993, the MS degree in 1997, and the Ph.D. degree in 2001 in mechanical engineering from the University of Zagreb, Croatia. He is an Assistant Professor at the Department of Control Engineering at the Faculty of Mechanical Engineering and Naval Architecture on the University of Zagreb. His research interests include control of dynamic systems, fluid power systems control, mechatronics and computer simulations.

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# 1 Introduction

Hydraulic systems are widely used actuators in industrial manufacturing plants and heavy-duty mobile systems because of their high power-to-weight ratio, high stiffness, fast response, self-cooling and high payload capability. Therefore, hydraulic actuators have widespread use in a variety of applications including manufacturing processes (machine tools, metal forming machines, injection molding machines, metal testing devices, etc.), aircraft control system (for actuation flight surfaces), robotic manipulators, mining, forestry, civil engineering and many others. There exist a great number of applications where hydraulic actuators are used with simple valves of the on/off type, which are operated manually or electronically using solenoid valves. For less demanding applications this form of control is satisfactory and their employment will be continued in the future. Integration of electronics and hydraulics into electro-hydraulic servo systems created a solution that promise unique application opportunities unmatched by other drive technologies, especially in the areas of large actuation forces. In control applications where high dynamic performances are important it is common to achieve the actuator in a servo loop stating a feedback transducer and electronically realized controller.

Different types of control components for flow or pressure control exist including manual (or solenoid) on/off control valves, proportional directional control valves and servo valves. The distinction between proportional and servo technologies is not clear defined and the selection of a control valve for a particular application is dependent on the overall control system requirements, the terms to use of control system, the requirements for accuracy and repeatability and of course the hardware cost. Proportional valves usually have slower response, less repeatability and they normally do not have an internal feedback for spool position correction within the valve. Proportional valves are traditionally used in open-loop control. However, technological improvements of proportional valves reduce the difference between the overall performances of proportional and servo valves, so that proportional valves can be an economical solution in many closed-loop applications, which have been dominated by costly servo valves. The proportional and servo valves as the control components are used to regulate the oil flow rate to the actuators, and thereby their motion. Both linear (hydraulic cylinders) and rotary (hydraulic motors) actuators are used in servo loops and add to the flexibility of hydraulic power systems.

The control valve adjusts the oil flow to move the actuator until the desired linear position (or rotary angle) is reached, or in a similar way adjusts the oil pressure until the desired force (or torque) is achieved, measured by a suitable transducer. Thus, the electro-hydraulic actuators enable realization of both position/speed-controlled and pressure/force-controlled drives.

Hydraulic servo systems have many uncertainties, which are consequences of physical characteristics, disturbances and load variations. The control valves are the major source of nonlinearities in control systems due to the phenomena such as nonlinear pressure-flow characteristics through the orifice inside the valve body, dead zone (valve's characteristic near null), hysteresis in flow-gain characteristic due to magnetic properties of the solenoid coil and friction [1]. Because of the analytical complexity involved it is very challenging task to obtain an accurate mathematical model of hydraulic actuator controlled with proportional or servo valve, which could satisfactorily describe the behavior of the control process. Therefore, computer simulations can be used to show the form of the control system's behavior, effects of parameter changes and for controller design procedure.

# 2 Modeling the hydraulic system

The hydraulic actuator is a device which converts hydraulic energy into mechanical energy and produce linear or rotary movement. The physical model of an electro-hydraulic servo system for both translational and rotational motion control is shown in Fig. 1. The motion of the actuator is controlled by a proportional valve, where higher control input voltage can produce larger valve oil flow from the proportional valve and fast translational motion of the cylinder or fast rotational motion of the motor. The mathematical model of the hydraulic system is obtained from the model of the hydraulic proportional valve dynamic, then by applying the flow continuity through the valve orifice, then by analyzing the pressure behavior in the actuator chambers, and finally, by applying the force/torque balance equation for the actuator.

Proportional valve integrates the ability of advanced electronic control with the high power output of hydraulic actuation. The solenoids of the proportional valve convert an electrical signal into a proportional movement of the valve's sliding spool element. In order to represent proportional valve dynamics through a wider frequency range a second-order dynamical term can be used, which gives the relation between the spool position $y_{\mathrm{v}}$ and the input current driving the valve $i$ in the transfer function form as follows:

$$\frac{y_{\mathrm{v}}(s)}{i(s)} = \frac{K_{\mathrm{v}}\,\omega_{\mathrm{v}}^2}{s^2 + 2\,\zeta_{\mathrm{v}}\,\omega_{\mathrm{v}}\,s + \omega_{\mathrm{v}}^2} \qquad (1)$$

where $K_{\mathrm{v}}$ is the proportional valve gain, $\omega_{\mathrm{v}}$ is the undamped natural frequency and $\zeta_{\mathrm{v}}$ is the damping coefficient of the proportional valve.

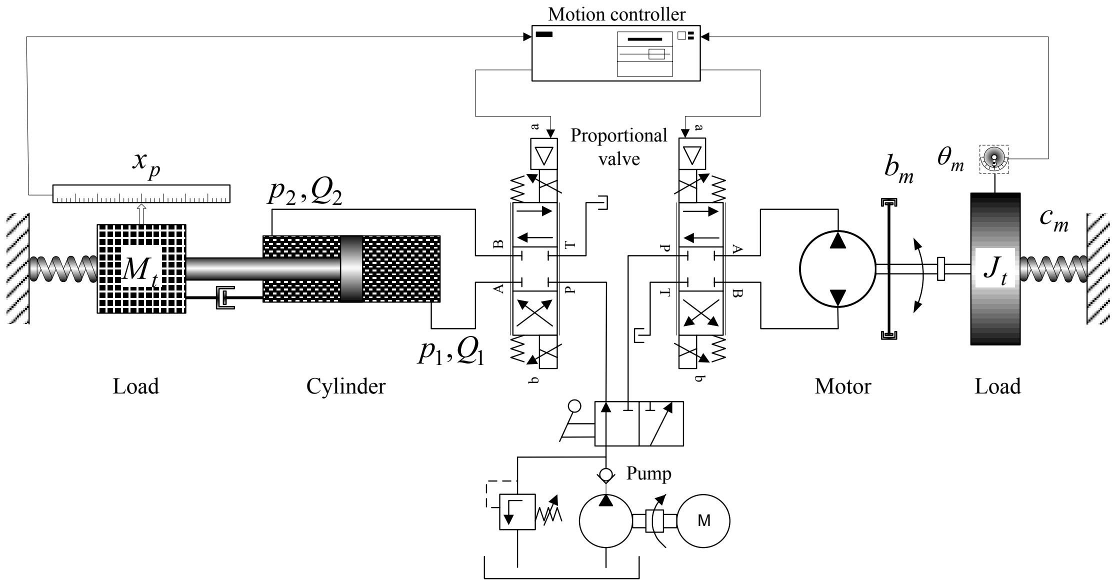


Fig. 1 The structure of the hydraulic system

In practice, because the dynamics of the valve is usually much faster than the dynamics of the actuator and load, the mathematical model of the valve dynamic is very often represented as an equivalent first-order dynamical term, or even the valve dynamic is neglected.

The fluid flow entering and exiting the actuator chambers denoted by $Q_1$ and $Q_2$ are derived from the application of flow continuity through the orifice and may be defined as [2]:

$$Q_1(y_v, p_1) = y_v \sqrt{|\Delta p_P|}\,\mathrm{sgn}(\Delta p_P),$$
$$\text{where } \Delta p_P = \begin{cases} p_s - p_1 & \text{for} \quad y_v \ge 0 \\ p_1 - p_a & \text{for} \quad y_v < 0 \end{cases} \tag{2}$$
$$Q_2(y_v, p_2) = -y_v \sqrt{|\Delta p_R|}\,\mathrm{sgn}(\Delta p_R),$$
$$\text{where } \Delta p_R = \begin{cases} p_2 - p_a & \text{for} \quad y_v \ge 0 \\ p_s - p_2 & \text{for} \quad y_v < 0 \end{cases}$$

where $p_s$ is the supply pressure, $p_a$ is the tank pressure (assuming atmospheric), $p_1$ and $p_2$ are the pressures in the actuator chambers. It is assumed that the valve orifice coefficients are included in the proportional gain $K_v$ of the valve spool dynamics. If we suppose that the flows entering and exiting the actuator chambers are equal in respect to the amount but have the opposite direction the following can be written:

$$Q_1(y_v, p_1) = -Q_2(y_v, p_2)\,. \tag{3}$$

If internal and external leakages are neglected, hydraulic pressure behavior for a compressible fluid volume is given by the differential equation:

$$\frac{V}{B}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\mathrm{d}V}{\mathrm{d}t} = Q \tag{4}$$

where $B = -V\dfrac{\mathrm{d}p}{\mathrm{d}V}$ is the fluid bulk modulus. From the above expression the piston and rod side actuator pressures may be written as follows:

$$\begin{aligned} \dot{p}_1 &= \frac{B}{V_1}(Q_1 - \dot{V}_1) \\ \dot{p}_2 &= \frac{B}{V_2}(Q_2 - \dot{V}_2) \end{aligned} \tag{5}$$

In the above equations $V_1$ and $V_2$ and their derivatives are defined as:

| Linear actuator: | Rotary actuator: |
|---|---|
| $V_1 = V_0 + A_1 x_p$ | $V_1 = V_0 + D_m \theta_m$ |
| $\dot{V}_1 = A_1 \dot{x}_p$ | $\dot{V}_1 = D_m \dot{\theta}_m$ |
| $V_2 = V_0 - A_2 x_p$ | $V_2 = V_0 - D_m \theta_m$ |
| $\dot{V}_2 = -A_2 \dot{x}_p$ | $\dot{V}_2 = -D_m \dot{\theta}_m$ |

where $V_1$ and $V_2$ are the total fluid volumes in the two actuator chambers, $V_0$ is the average contained volume of each actuator chamber, $A_1$ and $A_2$ are the annulus area of the piston and rod side of the cylinder, $x_p$ is the position of the cylinder, $D_m$ is the

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volumetric displacement of the motor and $\theta_{\mathrm{m}}$ is the angular position of the motor shaft.

From the force/torque balance equation for the actuator it can be written:

$$\begin{aligned} \ddot{x}_{\mathrm{p}} &= \frac{1}{M_{\mathrm{t}}}(p_1 A_1 - p_2 A_2 - b\,\dot{x}_{\mathrm{p}} - c\,x_{\mathrm{p}} - F_{\mathrm{L}}) \\ \ddot{\theta}_{\mathrm{m}} &= \frac{1}{J_{\mathrm{t}}}(p_1 D_{\mathrm{m}} - p_2 D_{\mathrm{m}} - b_{\mathrm{m}}\,\dot{\theta}_{\mathrm{m}} - c_{\mathrm{m}}\,\theta_{\mathrm{m}} - T_{\mathrm{L}}) \end{aligned} \tag{6}$$

where $M_{\mathrm{t}}$ is the total mass of the piston and the load, $J_{\mathrm{t}}$ is the total inertia of motor and load, $b$ and $b_{\mathrm{m}}$ are the viscous damping coefficients, $c$ and $c_{\mathrm{m}}$ are the coefficients of load stiffness, $F_{\mathrm{L}}$ denotes a constant externally applied load on the cylinder and $T_{\mathrm{L}}$ is the external torque applied to the motor.

## 3 System simulation

The use of system modeling and simulation enables the generation of numerical values for the interest system variables and assists in analysis of the system. The simulation model is represented by a block diagram in which relationships between system variables are included. Each block in the simulation diagram represents a dynamic behavior in the actual system and each line represents an information or energy signal. The diagram shows the interconnections between the digital computer for setting input to the model and controlling the system, proportional valve as a control component, actuator and measuring system. Using the mathematical model based on proportional valve dynamics, mass flow rate and the time rate of change of the cylinder pressure and equation of motion, the model is put together in Matlab's Simulink block diagram format shown in Fig. 2. The simulation results for both directions of the cylinder motion from the mid-stroke position [3], as well as the motor angular position [4] are shown in Fig. 3. Simulations are performed using the following system parameters listed in Tab. 1. Time-response characteristics of both control systems were investigated by applying step reference input. The time-response plots for the actuator position, the actuator velocity, the solenoid current, the valve spool stroke, the fluid flow entering and exiting the actuator chambers and the pressure in the actuator chambers are presented. For given numerical values the considered systems are observed to yield similar response characteristics.

Tab. 1 Values of system parameters

| Par. | Value | Par. | Value |
|---|---|---|---|
| $\omega_{\mathrm{v}}$ | 113 rad/s | $V_0$ | $1.4228\cdot10^{-4}$ |
| $\zeta_{\mathrm{v}}$ | 0.4 | $B$ | $1350\cdot10^{6}$ Pa |
| $K_{\mathrm{v}}$ | $5.5\cdot10^{-7}$ m/mA | $M_{\mathrm{t}}$ | 100 kg |
| $K_{\mathrm{m1}}$ | 33.33 V/m | $b$ | 455 Ns/m |
| $K_{\mathrm{m2}}$ | 1 V/rad | $c$ | $10^5$ N/m |
| $K_{\mathrm{R}}$ | 0.25 V/m | $D_{\mathrm{m}}$ | $25.4\cdot10^{-6}$ m$^3$/rad |
| $p_{\mathrm{s}}$ | $150\cdot10^5$ Pa | $J_{\mathrm{t}}$ | $1.56\cdot10^{-3}$ kgm$^2$ |
| $p_{\mathrm{a}}$ | $10^5$ Pa | $b_{\mathrm{m}}$ | 0.5 Nms/rad |
| $A$ | $9.485\cdot10^{-4}$ m$^2$ | $c_{\mathrm{m}}$ | 150 Nm/rad |

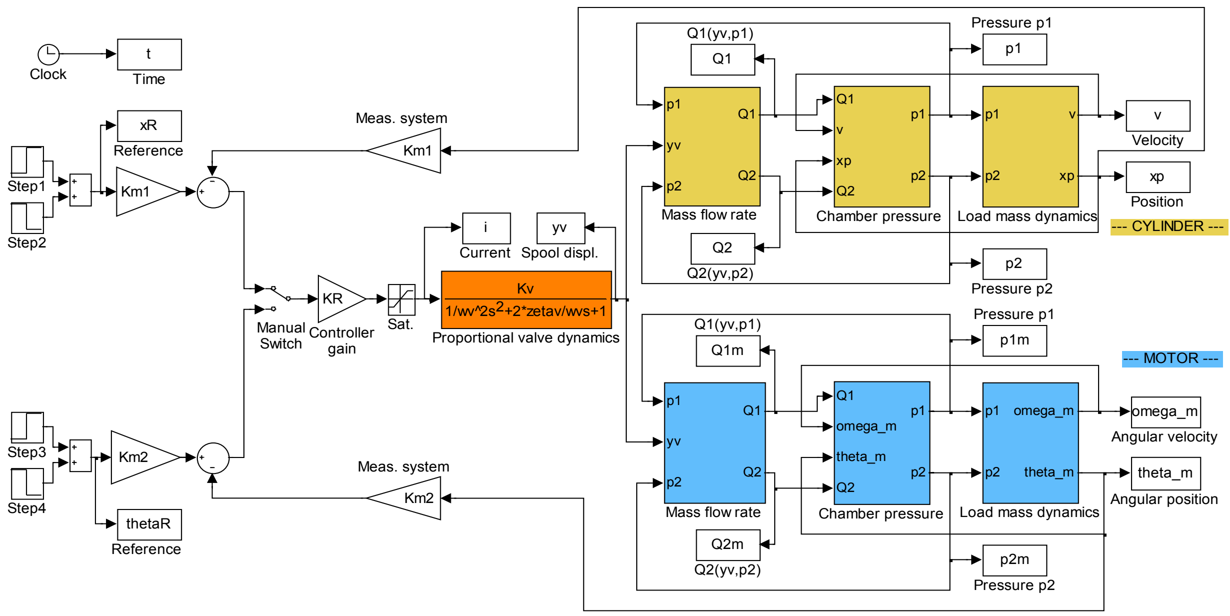


Fig. 2 Simulation block diagram of the control system

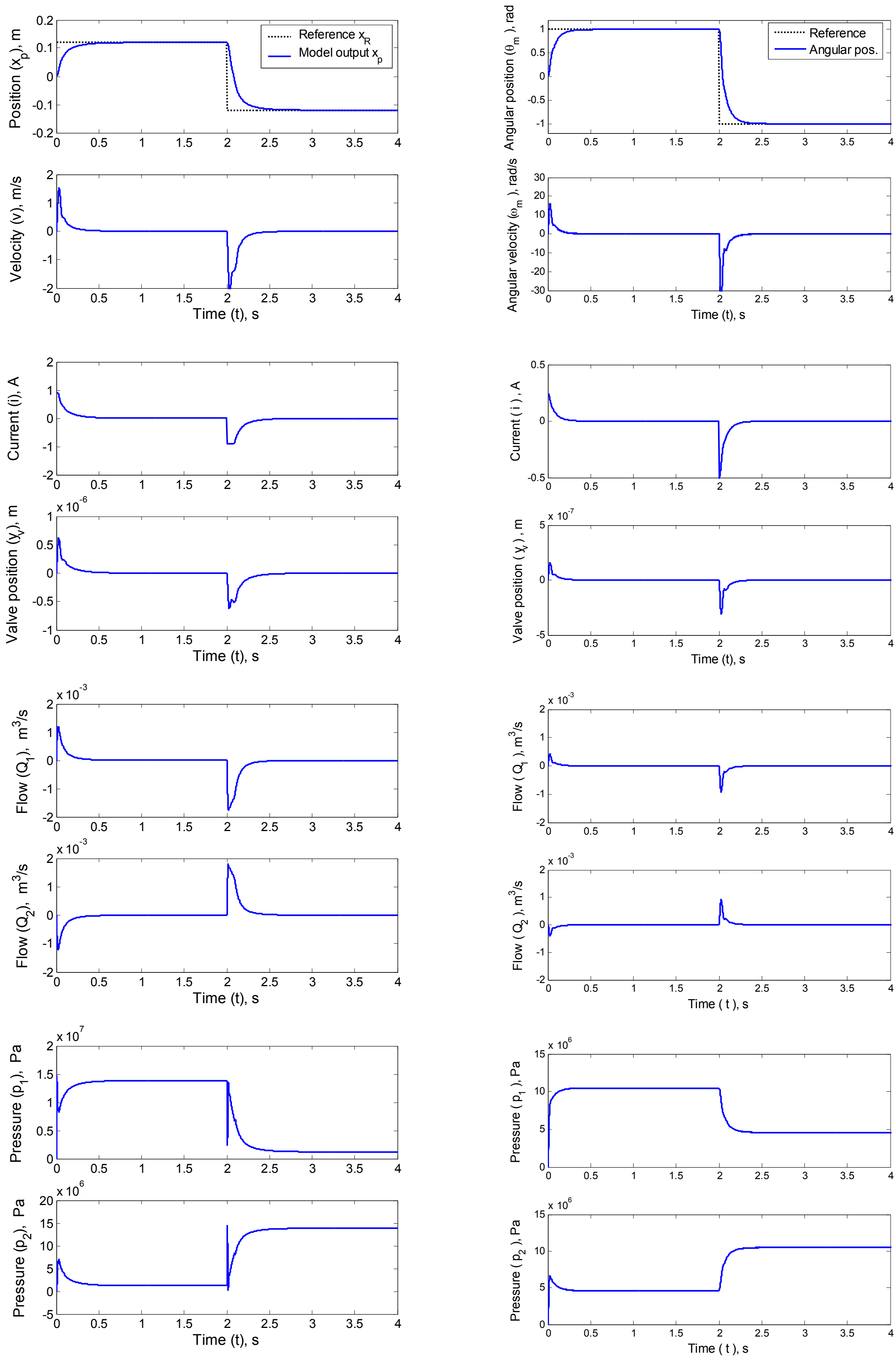


Fig. 3 Simulated responses for the step change in the reference signal

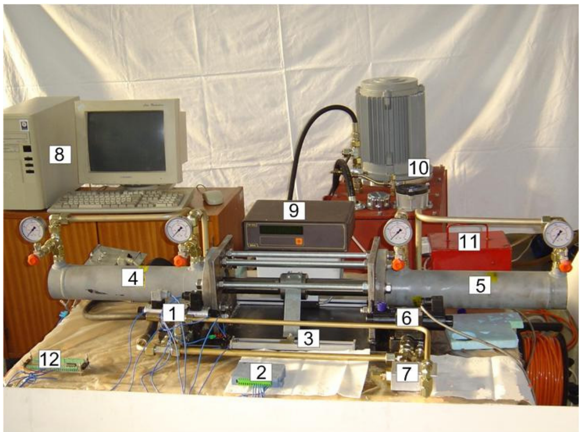


Fig. 4 Experimental model for translational motion control

A photo of our realized experimental test model for translational motion control is shown in Fig. 4. The structure of the system consists of the main control system shown at the left-hand side of the figure and the load inertia (disturbance system) shown at the right-hand side of the figure. The main control system consists of a main cylinder (4), which is controlled by using a three-way proportional directional control valve (1), equipped with the electronic control card (2) that can supply the cylinder with hydraulic fluid. The disturbance is generated by using a directional control valve (6) and a load cylinder (5), which produces a reaction force. This force is equivalent to the product of the piston area and the controlled pressure which is generated by using a pressure control valve (7). The piston position of the main cylinder along its stroke is measured by using a linear potentiometer (3), which is attached to the actuator, and is also visible on a numerical display unit (9). The feedback control algorithms are implemented in a control computer (8) equipped with a data acquisition card. The hydraulic power supply (10) utilizes a gear pump to deliver the fluid to the system.

The controller design based on a linearized control-oriented model and the experimental results for the position control of the hydraulic cylinder for both loaded and unloaded drive can be found in [5], and details of the closed-loop control system realization in order to achieve precise positioning are not subject of this paper.

## 4 Conclusion

Hydraulic systems play an important role in many industries, such as machine tools, aerospace, robotics. The demanding market place for hydraulic systems calls for such systems to be more cost effective, have better performance, more reliable, safer and user friendly. During the last decades, hydraulic components suffered a great evolution towards microprocessor controlled electro-hydraulic devices, which allow them to improve the dynamic performances and enhance new control possibilities. In control systems where a fast dynamic response and a high precision are very important requirements, the computer simulations can be a cheap and an effective way to investigate some advanced control schemes. Modeling and simulation of hydraulic systems has received attention of many research institutions and both general purpose tools and special purpose software packages have been produced. However, specialized packages commonly use private modeling methodologies and limited component libraries with a set of predefined hydraulic modules that can be combined easily with electrical or mechanical components, providing a comprehensive model for complex electro-hydraulic applications [6].

The developed simulation model can be used to predict the performance of the control system and to provide insights for improving the behavior of the system. To evaluate the dynamic models of the control systems the equations were solved using Matlab/Simulink™ simulation software and the parameter values given in Tab. 1. The application of advanced control strategies on real hydraulic systems is a difficult task due to the problems of reproducing the system's working conditions. The advantages of having a simulation model for a hydraulic control system is that it enables users to easily evaluate the system behavior under different operating conditions and changes in system parameters. Using the numerical model, design changes can be quickly analyzed and also the simulated environment is a low-cost way to test developed control algorithms. Such a simulation model can be used as a guideline to do rapid prototyping, a process that allow automatic generation source code which may be compiled and run on the target processor directly from the block diagram modeling environment.

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